

Spatial susceptibility modulation and controlled unidirectional reflection amplification via four-wave mixing

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Abstract: Control of unidirectional light propagation is of paramount importance to optical signal processing and optical communication. Especially, the amplified optical signal can isolate noise well that may provide more applications. In this work, we propose a dynamically modulated regime to realize unidirectional reflection amplification in a short and dense uniform atomic medium, and all atoms are driven into four-level double- Λ type by two coupling fields with linearly varied intensities along x direction and two weak probe fields. Based on four-wave mixing resonance and the broken spatial symmetry, the complete nonreciprocal reflection (unidirectional reflection) can be amplified with reflectivity more than 2.0, even to 6.0. In addition, the width, height, and position of the unidirectional reflection bands can be tunable. Thus, our regime is feasible and may inspire further applications in all-optical networks that require controllable unidirectional light amplification.

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1. Introduction

Optical non-reciprocity is widely used to design novel photonic devices, such as all-optical diodes and chip isolators [1,2], which can isolate noise, stabilize signals on integrated chips and protection of sensitive laser equipment from unwanted reflections [3–6]. Efficient and stable integrated chips are the key in quantum computing and information processing [7–11]. Therefore, it is of great significance to carry out research on non-reciprocal light propagation to promote current quantum technology. The conventional route to optical nonreciprocity is mainly based on the Faraday effect in magneto-optical medium [12–14], which leading to the materials' permittivity becoming an asymmetric anisotropic tensor. However, the bulky magnets is always required in order to achieve high external magnetic field, in addition, the inevitable loss inherent to magneto-optical materials are also significant limiting factors, especially at optical frequencies. Thus, the mechanism depending on magneto-optical Faraday effect to realize symmetry-breaking effect is incompatible with integrated circuit technology.

Recent researches show that optical nonreciprocity of non-magnetic has made great progress. In chiral quantum optical systems, nonreciprocity can be realized by asymmetric coupling of atomic internal states [15–18]; Such a nonreciprocal light manipulation has achieve important results in optomechanical systems, that on a few photon or even single-photon level [19–22]; It has also been widely studied in nonlinear optical systems, *e.g.*, a parity-time (PT) symmetric or antisymmetric system as standing-wave coupling fields are applied in a one-dimensional atomic lattice [23–25], or a moving atomic lattices which based on time reversal symmetry breaking [26–28], even in the hot atom system with considering Doppler shifts [29]. It is then justified, the above physical systems have made significant progress in studying optical nonreciprocity. However, the challenge in these schemes is the requirement of precise modulation, including achieving a balance of gain and loss within a single period, precisely arranging the spatial

coupling fields, and the unidirectional propagation that probe light incident from opposite side completely suppressed is difficult to achieve.

Alternatively, Horsley *et al.* found that asymmetric and unidirectional reflection can be achieved in homogeneous continuous medium, which requires the real and imaginary part of refractive index, polarizability or dielectric constant of the plane electromagnetic wave obeying the spatial Kramers-Kronig (KK) relation [30,31]. Based on the KK relation, the realization of omnidirectional perfect absorber and medium with no transmission can be promised in different inhomogeneous media [32–35]. In addition, the unidirectional reflection can be modulated by suitable design of controlled Rydberg atom or the intensities of coupling fields, that making the probe susceptibility satisfy the spatial KK relation [36,37]. It is worth stress that the spatial KK relations do not require any definite symmetry under space inversion or time reversal, and whatever the angle of incident light, so it is more simple and controllable in experiment. However, the main difficulty in implementing this scheme lies in the need for skillfully constructing spatial-KK media, and improving the low reflectivity of unidirectional reflection. The theoretical scheme of nonreciprocal amplification has been proposed in optomechanical systems [38–40], non-Hermitian time-floquet systems [41], in Josephson circuits dominated by microwave [42,43]. Recently, an irreversible transmission-light amplification has been demonstrated theoretically and experimentally, based on the Doppler effect in the hot atomic system [44]. It is worth emphasizing that optical non reciprocity has been widely studied in coherent cold atomic systems. However, the nonreciprocal amplification in thus systems remains to be studied.

In this paper, we propose a theoretical model of four-level atoms via four-wave mixing (FWM), and accordingly realize amplified unidirectional reflections of two probe fields based on electromagnetically induced transparency (EIT) using rubidium (Rb) atoms homogeneously distributed. The interaction of four laser beams caused nonlinear optical phenomena can induce probe light amplified in the EIT window [45], and the unidirectional reflection is due to the disruption of spatial symmetry by reasonably setting the densities of coupling fields accompanied by the corresponding variation of the spatial position [46]. It needs to be emphasized, the amplified optical signal can prevent the external noise, so the unidirectional amplification is of paramount importance to one-way optical communication.

2. Model and equations

The schematic of our approach to FWM amplification is depicted in Fig. 1(a). There the cold ^{87}Rb atoms in the cell extending from $x = 0$ to $x = L$, driven by two weak probe fields of amplitudes (frequencies) $\mathbf{E}_{pi}$ (ω_{pi}) and two strong coupling fields of amplitudes (frequencies) $\mathbf{E}_{ci}$ (ω_{ci}) with $i = 1, 2$. All atoms are driven into four-level double- Λ configuration, characterized by the Rabi frequency (detuning) Ω_{c1} ($\Delta_{c1} = \omega_{c1} - \omega_{31}$) for the transition $|1\rangle \longleftrightarrow |3\rangle$, Ω_{c2} ($\Delta_{c2} = \omega_{c2} - \omega_{42}$) for the transition $|2\rangle \longleftrightarrow |4\rangle$, Ω_{p1} ($\Delta_{p1} = \omega_{p1} - \omega_{32}$) for the transition $|2\rangle \longleftrightarrow |3\rangle$ and Ω_{p2} ($\Delta_{p2} = \omega_{p2} - \omega_{41}$) for the transition $|1\rangle \longleftrightarrow |4\rangle$ respectively. Here, the Rabi frequency $\Omega_{c1} = \mathbf{E}_{c1} \cdot \mathbf{d}_{31}/2\hbar$, $\Omega_{c2} = \mathbf{E}_{c2} \cdot \mathbf{d}_{42}/2\hbar$, $\Omega_{p1} = \mathbf{E}_{p1} \cdot \mathbf{d}_{32}/2\hbar$ and $\Omega_{p2} = \mathbf{E}_{p2} \cdot \mathbf{d}_{41}/2\hbar$. The matrix element $\mathbf{d}_{ij} = \langle i | \mathbf{d} | j \rangle$ is used to denote the dipole moment for transition $|i\rangle \longleftrightarrow |j\rangle$. The homogeneous distribution of atoms is displayed in Fig. 1(c), the probe lights travel along the x -axis and the control fields travel vertical x -axis. In further, we assume the control fields are linearly varied in intensity along the x direction, e.g., by a neutral density filter. Thus, the Rabi frequencies can be expressed as $|\Omega_{c1}(x)|^2 = |\Omega_{c10}|^2 (k_1 x + b_1)$ and $|\Omega_{c2}(x)|^2 = |\Omega_{c20}|^2 (k_2 x + b_2)$. The efficient coupling between the four waves may occur only when energy and momentum are both conserved, this requires a balance FWM process $\Delta_{c1} - \Delta_{p1} + \Delta_{c2} - \Delta_{p2} = 0$, and a phase matching $\mathbf{k}_{c1} - \mathbf{k}_{p1} + \mathbf{k}_{c2} - \mathbf{k}_{p2} = 0$ [47,48]. Note, phase matching requires precise design, e.g., two coupling fields are counter-propagating vertical x -axis, and the two probe fields are also counter-propagating along x -axis with the small angles as shown in Fig. 1(d). Last but not least,

it is worth to emphasize that the Doppler broadening can be ignored safely in our cold atomic sample.

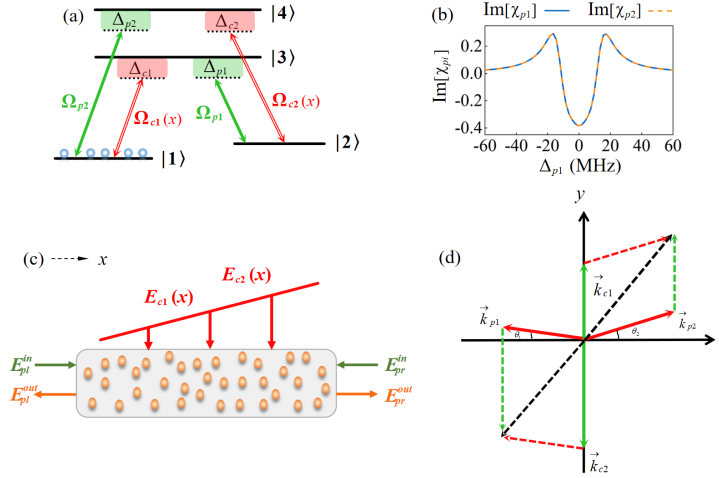


Fig. 1. (a) Energy level diagram of a four-level system driven by two weak probe-fields and two strong coupling fields. (b) The imaginary parts of two probe susceptibilities v.s. detuning Δ_{p1} . (c) Diagram of the homogeneous atomic medium illuminated by two coupling fields $E_{c1}(x)$ and $E_{c2}(x)$ vertical x axis, and two probe fields travel along the x direction. (d) Diagram of wave vectors of four laser fields.

With the electric-dipole and rotating-wave approximations, the atom-field Hamiltonian interaction in matrix form is

$$H_I = \begin{bmatrix} 0 & 0 & -\Omega_{c1}^*(x) & -\Omega_{p2}^* \\ 0 & \Delta_{c1} - \Delta_{p1} & -\Omega_{p1}^* & -\Omega_{c2}^*(x) \\ -\Omega_{c1}(x) & -\Omega_{p1} & \Delta_{c1} & 0 \\ -\Omega_{p2} & -\Omega_{c2}(x) & 0 & \Delta_{p2} \end{bmatrix}. \quad (1)$$

Then, we can substitute Eq. (2) into the master equation of lossless quantum mechanics $i\frac{\partial \rho}{\partial t} = [H_I, \rho]$ with H_I , the equations of motion of the density matrix are written as:

$$\begin{aligned} \dot{\rho}_{11} &= i\Omega_{c1}^*(x)\rho_{31} - i\Omega_{c1}(x)\rho_{13} + i\Omega_{p2}^*\rho_{41} - i\Omega_{p2}\rho_{14} + \Gamma_{31}\rho_{33} + \Gamma_{41}\rho_{44} \\ \dot{\rho}_{12} &= [i(\Delta_{c1} - \Delta_{p1}) - \gamma_{12}]\rho_{12} + i\Omega_{c1}^*(x)\rho_{32} - i\Omega_{p1}\rho_{13} + i\Omega_{p2}^*\rho_{42} - i\Omega_{c2}\rho_{14} \\ \dot{\rho}_{13} &= (i\Delta_{c1} - \gamma_{13})\rho_{13} - i\Omega_{c1}^*(x)\rho_{11} - i\Omega_{p1}^*\rho_{12} + i\Omega_{c1}^*(x)\rho_{33} + i\Omega_{p2}^*\rho_{43} \\ \dot{\rho}_{14} &= (i\Delta_{p2} - \gamma_{14})\rho_{14} - i\Omega_{p2}^*\rho_{11} - i\Omega_{c2}^*(x)\rho_{12} + i\Omega_{c1}^*(x)\rho_{34} + i\Omega_{p2}^*\rho_{44} \\ \dot{\rho}_{22} &= i\Omega_{p1}^*\rho_{32} - i\Omega_{p1}\rho_{23} + i\Omega_{c2}^*(x)\rho_{42} - i\Omega_{c2}(x)\rho_{24} + \Gamma_{32}\rho_{33} + \Gamma_{42}\rho_{44} \\ \dot{\rho}_{23} &= (i\Delta_{p1} - \gamma_{23})\rho_{23} - i\Omega_{c1}^*(x)\rho_{21} - i\Omega_{p1}^*\rho_{22} + i\Omega_{p1}\rho_{33} + i\Omega_{c2}^*(x)\rho_{43} \\ \dot{\rho}_{24} &= (i\Delta_{c2} - \gamma_{24})\rho_{24} - i\Omega_{p2}^*\rho_{21} - i\Omega_{c2}^*(x)\rho_{22} + i\Omega_{p1}^*\rho_{34} + i\Omega_{c2}^*(x)\rho_{44} \\ \dot{\rho}_{33} &= i\Omega_{c1}(x)\rho_{13} - i\Omega_{c1}^*(x)\rho_{31} + i\Omega_{p1}\rho_{23} - i\Omega_{p1}^*\rho_{32} - \Gamma_{32}\rho_{33} - \Gamma_{31}\rho_{33} \\ \dot{\rho}_{34} &= [i(\Delta_{p2} - \Delta_{c1}) - \gamma_{34}]\rho_{34} + i\Omega_{c1}(x)\rho_{14} - i\Omega_{p2}^*\rho_{31} + i\Omega_{p1}\rho_{24} - i\Omega_{c2}^*(x)\rho_{32} \end{aligned} \quad (2)$$

According to the definition, the density matrix element $\rho_{ij} = \langle i | \rho | j \rangle = \langle i | \Psi \rangle \langle \Psi | j \rangle = C_j C_i^*$ is the product of the probability amplitude of the population between states $|i\rangle$ and $|j\rangle$, and it

indicates the state of coherence occurring in the system. Additionally, $\gamma_{ij} = (\Gamma_i + \Gamma_j)/2$ denotes the complex coherence dephasing rate on transitioning $|i\rangle \longleftrightarrow |j\rangle$, with population decay rates $\Gamma_i = \Sigma_k \Gamma_{ik}$ and $\Gamma_j = \Sigma_k \Gamma_{jk}$; $k = 1, 2, 3$ and 4 describes the inevitable dissipation within the system and $\Gamma_{31} = \Gamma_{32} = \Gamma_{41} = \Gamma_{42} = \Gamma$. The above equations constrained by $\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1$ and conjugate conditions $\rho_{ij} = \rho_{ji}^*$. Under the steady-state condition $\dot{\rho}_{ij} \approx 0$, We can obtain ρ_{32} and ρ_{41} which governed by probe detuning Δ_p and position x by numerical solution.

Correspondingly, the complex susceptibility of the probe field $\chi_{pi}(x)$ ($i = 1, 2$) in this system yields

$$\begin{aligned}\chi_{p1}(\Delta_p, x) &= \text{Re}[\chi_{p1}(\Delta_p, x)] + \text{Im}[\chi_{p1}(\Delta_p, x)] \\ &= \frac{N_0 |\mathbf{d}_{32}|^2 \rho_{32}(\Delta_p, x)}{\varepsilon_0 \Omega_{p1}}, \\ \chi_{p2}(\Delta_p, x) &= \text{Re}[\chi_{p2}(\Delta_p, x)] + \text{Im}[\chi_{p2}(\Delta_p, x)] \\ &= \frac{N_0 |\mathbf{d}_{41}|^2 \rho_{41}(\Delta_p, x)}{\varepsilon_0 \Omega_{p2}}\end{aligned}\quad (3)$$

where, $\text{Re}[\chi_{pi}(\Delta_p, x)]$ and $\text{Im}[\chi_{pi}(\Delta_p, x)]$ represent the real and imaginary parts of susceptibility with the atomic density N_0 is a constant, and the complex refractive index $n_{pi}(x) = \sqrt{1 + \chi_{pi}(\Delta_p, x)}$.

In the following, we should check the light transport features, which can be examined by directly adopting standard transfer-matrix method [49]. First of all, we provide the j th 2×2 unimodular transfer matrix $m_j(\Delta_p, x_j)$ by dividing the whole sample of length L into S thin layers, $j \in (1, S)$. With identical thickness $\delta = L/S$, but the susceptibilities exhibit slightly different which lead the various of transfer matrix

$$m_j(\Delta_p, x_j) = \frac{1}{t_j(\Delta_p, x_j)} \cdot \begin{bmatrix} (t_j(\Delta_p, x_j)^2 - r_j^r(\Delta_p, x_j)r_j^l(\Delta_p, x_j)) & r_j^l(\Delta_p, x_j) \\ -r_j^r(\Delta_p, x_j) & 1 \end{bmatrix} \quad (4)$$

with the corresponding right and left reflection- and transmission-complex amplitudes $r_j^{r,l}(\Delta_p, x_j)$ and $t_j^r(\Delta_p, x_j) = t_j^l(\Delta_p, x_j) = t_j(\Delta_p, x_j)$, respectively, determined by the complex refractive index $n_p(\Delta_p, x_j)$. After that, we can write the total transfer matrixes of j layers

$$M^l(\Delta_p, j\delta) = m_1(\Delta_p, x_1) \times \cdots \times m_j(\Delta_p, x_j), \quad (5)$$

$$M^r(\Delta_p, j\delta) = m_S(\Delta_p, x_S) \times \cdots \times m_j(\Delta_p, x_j). \quad (6)$$

Note that $M^l(\Delta_p, j\delta)$ is multiplied from left to right by layers, and $M^r(\Delta_p, j\delta)$ is multiplied from right to left. Thus, the reflectivity of two probe fields at j th layer that incidents from the left-side and right-side are

$$R_{pi}^l(\Delta_p, j\delta) = \left| r_{pi}^l(\Delta_p, j\delta) \right|^2 = \left| \frac{M_{(12)}^l(\Delta_p, j\delta)}{M_{(22)}^l(\Delta_p, j\delta)} \right|^2, \quad (7)$$

$$R_{pi}^r(\Delta_p, j\delta) = \left| r_{pi}^r(\Delta_p, j\delta) \right|^2 = \left| \frac{M_{(12)}^r(\Delta_p, j\delta)}{M_{(22)}^r(\Delta_p, j\delta)} \right|^2, \quad (8)$$

with the complex amplitudes $r_{pi}^{l,r}(\Delta_p, j\delta)$ and $t_j(\Delta_p, j\delta)$ determined by the transfer matrixs can be clearly seen by Eqs. (7) and (8). When $j = S$ the reflectivities at both ends of this finite atomic

sample lead to the following expressions

$$\begin{aligned} R_{pi}^l(\Delta_p, L) &= \left| r_{pi}^l(\Delta_p, L) \right|^2 = \left| \frac{M_{(12)}^l(\Delta_p, L)}{M_{(22)}^l(\Delta_p, L)} \right|^2 \\ R_{pi}^r(\Delta_p, L) &= \left| r_{pi}^r(\Delta_p, L) \right|^2 = \left| \frac{M_{(12)}^r(\Delta_p, L)}{M_{(22)}^r(\Delta_p, L)} \right|^2 \end{aligned} \quad (9)$$

That is $M_{(22)}^l(\Delta_p, L) = M_{(22)}^r(\Delta_p, L)$ and $M_{(21)}^l(\Delta_p, L) = M_{(12)}^r(\Delta_p, L)$ only at the ends of the sample, with the sample length $L = S\delta$. The spatial symmetry of susceptibility is destroyed due to the linearly variation of coupling field intensities, which leads to the nonreciprocity of left-side and right-side reflections. This asymmetric reflection can be expressed by the contrast factor:

$$C_{pi}(\Delta_p, L) = \left| \frac{R_{pi}^l(\Delta_p, L) - R_{pi}^r(\Delta_p, L)}{R_{pi}^l(\Delta_p, L) + R_{pi}^r(\Delta_p, L)} \right| \quad (10)$$

an important figure of merit to check the nonreciprocal reflection. Obviously, $0 \leq C_{pi}(\Delta_p, L) \leq 1$ in an optical response system. If and only if $C_{pi}(\Delta_p, L) = 0$, the left-side and right-side reflections are reciprocal. Otherwise, the left-side and right-side reflections are nonreciprocal. And when $C_{pi}(\Delta_p, L) = 1$, the unidirectional reflection can be realized.

3. Results and discussion

In this section, we will check the optical nonreciprocity in a probe gain system, and realize the amplified unidirectional reflections by linearly modulating the intensity of coupling fields. Firstly, we consider the simplest case that the two coupling fields have the same variation for $|\Omega_{c10}|^2 = |\Omega_{c20}|^2 = |\Omega_0|^2$, $k_1 = k_2 = k$ and $b_1 = b_2 = b$ are adjusted for convenience. Fig. 2 depicts the imaginary and real parts of probe susceptibility as a function of the position with three different probe detunings in panels (a) and (b), of the position and probe detuning in panels (c) and (d). It is easy to see in Figs. 2(a) and 2(b), the absorption spectral line ($\text{Im}[\chi_{pi}]$) exhibits an odd symmetry in the whole space, however, the dispersion spectral line ($\text{Re}[\chi_{pi}]$) exhibits an even symmetry only in part of the space. However, this peculiar symmetry is gradually lifted with the increasing of $|\Delta_{p1}|$, as shown in Figs. 2(c) and 2(d). Thus, the susceptibility exhibits the PT symmetry in part of the space described by $\chi_{pi}(x) = \chi_{pi}^*(-x)$ in our scheme. As we known, the PT symmetry is a critical characteristic for achieving nonreciprocity. We need stress that, if the susceptibility shows the PT symmetry, it satisfies the spatial Kramers-Kronig relation that is sufficient for zero reflection from one side [31]. In the following, we will check the unidirectional reflection in the condition that the susceptibility is not completely satisfy PT symmetry, and spatial Kramers-Kronig relation.

Next, we will examine the reflections of the incident probe light from right-side and left-side, respectively, with the parameters used in Fig. 2. As we can see, Fig. 3(a) [Fig. 3(b)] shows a large frequency region $\Delta_{p1} \in (-10 \text{ MHz}, 30 \text{ MHz})$ [$\Delta_{p1} \in (-30 \text{ MHz}, 10 \text{ MHz})$] of high reflection on the right-side R_{p1}^r [R_{p2}^r] of probe field E_{p1} [E_{p2}], with a sharp gain peak over than 2.0. It is noticeable that the peak of R_{p1}^r [R_{p2}^r] accompanied by $R_{p1}^l \rightarrow 0$ ($R_{p2}^l \rightarrow 0$), revealing a perfect unidirectional reflection. Which can also be clarified by the high-contrast factor C_{p1} (C_{p2}), an important figure of merit on the asymmetric reflection, could be up to 1.0 [see Fig. 3(c) and 3(d)]. In particular, we find that the probe reflections of right-side are much greater than 1. The physical reason can be expressed as follows. In our field-atom system, the probe fields can be amplified due to FWM resonance. That is, the absorptions of probe fields are negative for the imaginary parts of the probe susceptibilities $\text{Im}[\chi_{p1,2}] < 0$ [see Fig. 1(b)]. It is then justified that,

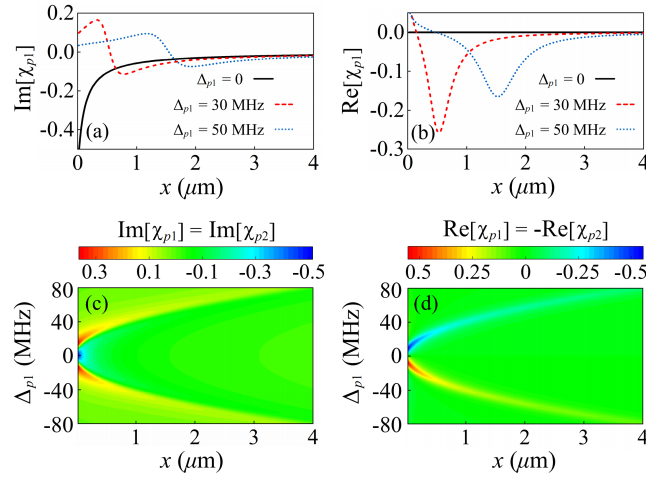


Fig. 2. (a) Imaginary and (b) real parts of susceptibility χ_{p1} v.s. position x with $\Delta_{p1} = 0$ (black solid line), 30 MHz (red dashed line) and 50 MHz (blue dashed line). (c) Imaginary and (d) real parts of susceptibility χ_{p1} v.s. both position x and detuning Δ_{p1} . Other parameters are $N_0 = 2 \times 10^{13} \text{ cm}^{-3}$, $\Omega_0 = 4 \text{ MHz}$, $k = 25 \text{ } \mu\text{m}^{-1}$, $b = 1.5$, $\Omega_{p1} = 0.01 \text{ MHz}$, $\Omega_{p2} = 0.01 \text{ MHz}$, $\Delta_c = \Delta_d = 0 \text{ MHz}$, $d_{14} = 2.0 \times 10^{-29} \text{ C}\cdot\text{m}$, $\Gamma_{41} = \Gamma_{42} = \Gamma_{31} = \Gamma_{32} = 6 \text{ MHz}$, medium length $L = 4 \text{ } \mu\text{m}$.

the unidirectional reflection amplification is the cooperation between satisfying PT symmetry and negative absorption of probe fields. In further, we plot the real and imaginary parts $\text{Im}[r_{pi}^r]$ and $\text{Re}[r_{pi}^r]$ ($\text{Im}[r_{pi}^l]$ and $\text{Re}[r_{pi}^l]$) of the reflection coefficients on the right-side r_{pi}^r (left-side r_{pi}^l), respectively, v.s. position x at $\Delta_p = \pm 15 \text{ MHz}$ (the peak of probe reflections), which determine the probe reflectivities in Figs. 3(e) and 3(f). It is clearly, the values of $\text{Im}[r_{pi}^l]$ and $\text{Re}[r_{pi}^l]$ are almost zero over the entire length of the medium, which leads to the light incidents on the left-side to be completely unreflected. However, the values of $\text{Im}[r_{pi}^r]$ and $\text{Re}[r_{pi}^r]$ are of harmonic oscillation and sensitive to the position x . It is of special interest that, when the absolute value of $\text{Im}[r_{pi}^r]$ reaches the maximum, the absolute value of $\text{Re}[r_{pi}^r]$ is at the minimum, and vice versa. Thus, the reflections on the right-side are always high, which is in good agreement with Figs. 3(a) and 3(b).

In Fig. 4, we analyze the tunability of amplified unidirectional reflections of two probe fields by the coupling detunings $\Delta_{c1} = \Delta_{c2} = \Delta_c$. In Figs. 4(a) and 4(b), it is found that the reflections of left-side are still be suppressed, with the reflectivity $R_{pi}^l < 0.06$ even to 0.02. Relative to the reflection bands for $\Delta_c = 0$, the two probe reflection bands of right-side become wide and high for $\Delta_c = -10 \text{ MHz}$, while become narrow and low for $\Delta_c = 10 \text{ MHz}$. Essentially, the frequency region of the gain peaks move irregularly with Δ_c . Therefore, we plot the right-side and left-side reflections of probe field E_{p1} v.s. detunings Δ_{p1} and Δ_c , in order to check the modulation of the gain peak by Δ_c in Figs. 4(c) and 4(d). It is obvious that the reflectivity on the left-side R_{p1}^l is below 0.1 in a large region of detuning Δ_c , especially at $\Delta_c \in (-20 \text{ MHz}, 0)$, as low as the minimum value of 0.02. Correspondingly, the reflectivity on the right-side R_{p1}^r can be amplified in this frequency region, and it is very high in a large region of detuning $\Delta_c \in (-40 \text{ MHz}, 0)$ with proper detuning Δ_{p1} . The physical interpretation is that the width of reflection band is decided by that of the EIT property, a weak probe field will not be absorbed in the frequency region of EIT window due to Fano interference, and it will either be transmitted or reflected even be amplified in the gain system. Thus, the reflection band is decided by that of the EIT property, which can be modulated by the detuning of coupling fields Δ_c . Thus, it can be seen that the width, height, and position of this probe reflection gain region may be controlled by modulating

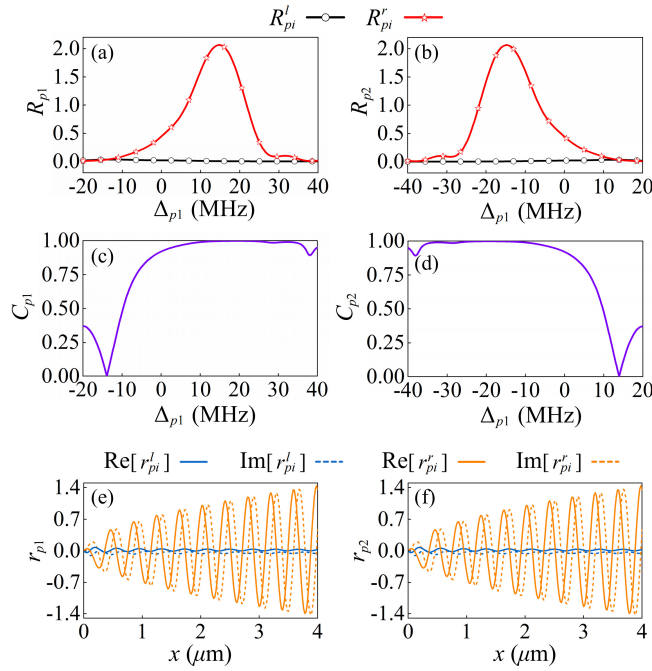


Fig. 3. Reflectivities on the right-side R_{pi}^r (red solid line) and left-side R_{pi}^l (black dotted line) of E_{p1} in (a) and E_{p2} in (b); the contrast factors of the left-side and right-side reflections C_{p1} and C_{p2} v.s. detuning Δ_{p1} in (c) and (d). Real and imaginary parts of reflection coefficients of the probe fields E_{p1} and E_{p2} v.s. position x that incidents from right-side (orange solid and dashed line) and left-side (blue solid and dashed line) in (e) and (f). Other parameters are the same as those in Fig. 2.

the matched coupling fields in frequency. Therefore, the amplified unidirectional reflection can be dynamically manipulated by adjusting the detuning of the coupling fields.

In Fig. 5, we further study the unidirectional reflection by the variation of the atomic density. It is clearly seen, both of two probe reflection bands on the right-side R_{pi}^r are increased with increasing atomic density. Although the two probe reflectivities that lights incident from the left-side can also be increased, the reflectivities are still very low $R_{pi}^l < 0.04$ shown corresponding to two insets. The physical insight is that the probe susceptibility linearly varies with the atomic density described by Eq. (3). It is worth emphasized that, in our regime, the probe reflections can be enhanced both by the increased atomic density and the FWM resonance. It means that the amplification of light in our system is more feasible.

Finally, We consider the different linear variations of two coupling fields how to modulate the unidirectional reflections in Fig. 6. Armed with previous analysis, the right-side reflections of two probe fields exhibit enormous amplification and the left-side reflections suffer strong absorption around resonance point, in the case of linearly increasing coupling intensity. To our particular interest, what will be happen if the intensity of the two control fields decreases linearly with x . There is a sharp gain peak of reflectivity on the left-side R_{pi}^l and significant absorption of reflectivity on the right-side R_{pi}^r , under near-resonance condition as shown in Figs. 6(a) and 6(b). This clearly demonstrates that the unidirectional reflection mainly depends on the spatial modulation of susceptibility. We further analyze the unidirectional amplification of probe reflectivity, corresponding to the condition that only one coupling field intensity increases linearly with x and the other one is constant, the reflections are shown in Figs. 6(c) and 6(d). It is

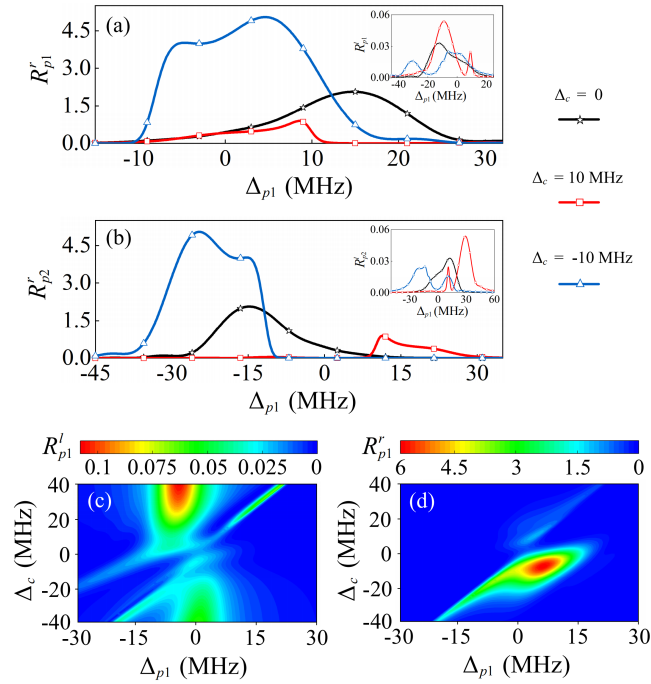


Fig. 4. Reflectivities on the right-side R_{pi}^r v.s. detuning Δ_{p1} and the two insets show the reflectivities on the left-side R_{pi}^l of probe field E_{p1} in (a) and E_{p2} in (b), with $\Delta_c = 0$ (black solid line), $\Delta_c = 10$ MHz (red solid line), $\Delta_c = -10$ MHz (blue solid line). Reflectivity on the left-side R_{pi}^l in (c) and right-side R_{pi}^r in (d) v.s. detuning Δ_{p1} and Δ_c of probe field E_{p1} . Other parameters are the same as those in Fig. 2.

clearly that the two unidirectional reflection bands decrease and move away from the resonance point $\Delta_{p1} = 0$ obviously. In particular, the probe reflectivity R_{p1}^r is sensitive to the coupling field

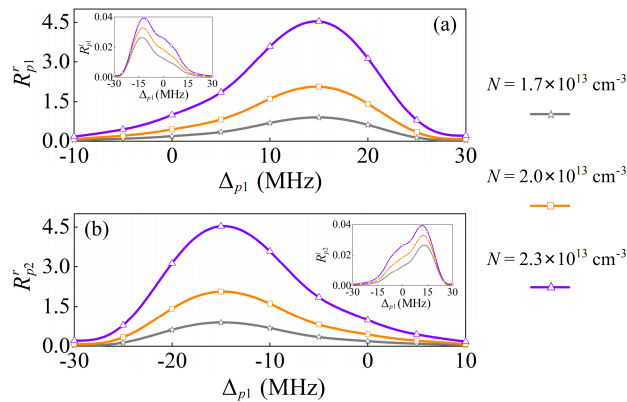


Fig. 5. Reflectivity on the right-side R_{pi}^r and left-side R_{pi}^l v.s. detuning Δ_{p1} of probe field E_{p1} in (a) and E_{p2} in (b), with $N_0 = 1.7 \times 10^{13} \text{ cm}^{-3}$ (grey solid line and star); $N_0 = 2.0 \times 10^{13} \text{ cm}^{-3}$ (orange solid line and square); $N_0 = 2.3 \times 10^{13} \text{ cm}^{-3}$ (purple solid line and triangle). Other parameters are the same as Fig. 2.

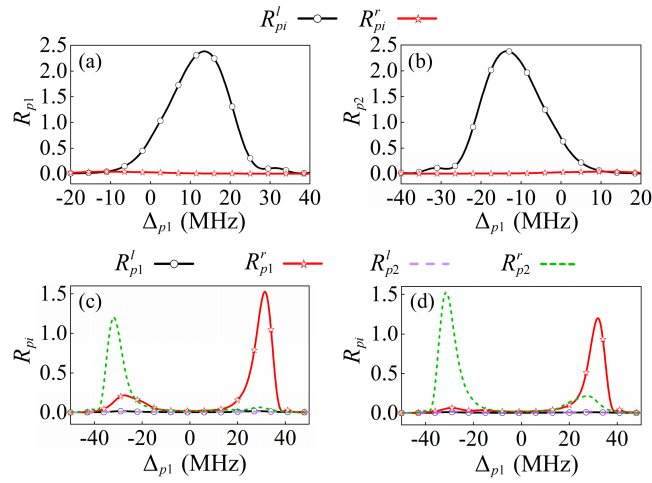


Fig. 6. Reflectivity on the right-side R_{pi}^r (red solid line) and left-side R_{pi}^l (black solid line) v.s. detuning Δ_{p1} of E_{p1} in (a) and E_{p2} in (b) with $k = -25 \mu\text{m}^{-1}$, $b = 101.5$; Reflectivity on the right-side R_{pi}^r and left-side R_{pi}^l v.s. detuning Δ_{p1} of E_{p1} and E_{p2} with $k_1 = 25 \mu\text{m}^{-1}$, $b_1 = 1.5$ and $\Omega_{c2}(x) = 25 \text{ MHz}$ in (c), with $\Omega_{c1}(x) = 25 \text{ MHz}$, $k_2 = 25 \mu\text{m}^{-1}$, $b_2 = 1.5$ in (d). Other parameters are the same as those in Fig. 2.

E_{c1} , and the probe reflectivity R_{p2}^r is sensitive to the coupling field E_{c2} . This is because the linear variation intensity of a single coupling field reduces the destructiveness of spatial symmetry of probe susceptibility, while another constant coupling field is not enough to obviously improve the gain of the probe light. Judging from this, our FWM gain regime with the linear variation intensity of two coupling fields can provide more flexible, that in modulating the unidirectional amplification of probe reflectivity.

Last but not least, we will explain the choice of initial parameters k and b in order to make readers better understand the setting of coupling field strength. The figure of the maximum of reflectivities R_{p1}^r and R_{p1}^l on the right- and left-side v.s. k at $\Delta_p = 15 \text{ MHz}$ [see the point in Fig. 3(a)] has been plotted as Fig. 7. It can be seen that the unidirectional reflection first increases

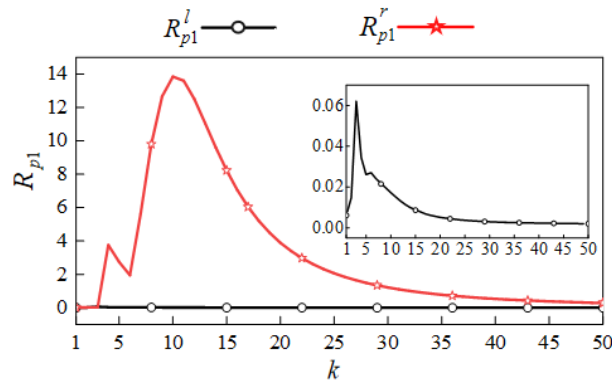


Fig. 7. Maximum of reflectivities on the right-side R_{p1}^r (red solid line and star) and left-side R_{p1}^l (black solid and ring) of E_{p1} v.s. coefficient k , at $\Delta_p = 15 \text{ MHz}$, the insert is reflectivities on the left-side R_{p1}^l . Other parameters are the same as Fig. 3.

and then decreases with the increase of slope k , and can be amplified only in the region $k \in (3 \mu\text{m}^{-1}, 32 \mu\text{m}^{-1})$. In addition, the unidirectional reflection is almost perfect with $k \geq 25 \mu\text{m}^{-1}$ shown by the insert. Here, we choose the initial parameter $k = 25 \mu\text{m}^{-1}$ to ensure that the perfect unidirectional reflection is amplified on the one hand, and to control the appropriate intensity of coupling fields to construct EIT mechanism on the other hand. EIT yields strong transparency for a weak probe field due to Fano interference. However, the Fano interference will disappear, if the coupling field is too strong, that is why the reflectivity R_{p1}^r is no longer amplified when $k \geq 32 \mu\text{m}^{-1}$. While $b = 1.5$ guarantees the intensity of coupling fields is strong enough to generate EIT when the probe light just enters the medium ($x = 0$). Because the intensity of coupling fields can be modulated by the parameters k and b , with the relation $|\Omega_c(x)|^2 = |\Omega_0|^2 (kx + b)$.

4. Conclusion

In summary, we have presented a study of unidirectional reflection amplification in a short and dense uniform atomic medium based on the FWM resonance, the atoms are driven into a four-level double-Lambda type by two probe fields and two strong coupling fields. It is worth noting that, the probe susceptibility is spatially modulated by the two coupling fields, for their intensity are linearly varying along the x direction in our scheme. For a reasonably strong modulation we show that one can obtain an interesting situation, wherein one direction probe light can be reflected and amplified, while in the opposite direction and at the same frequency, the reflectivity of probe light is almost completely suppressed. The physical essence is that the spatial symmetry of susceptibility is destroyed by the linear variation of coupling intensity, while the reflection is well enhanced by the FWM resonance. The main advantage of this regime is not only that it can realize unidirectional reflection amplification, but also that the width, height, and position of the reflection band is controllable. That is, our scheme is quite easy to be done in experiment and has more freedom on dynamic tunability and on-chip manipulation. Thus, the unidirectional amplifiers which can prevent the reverse flow of noise will represent a rather versatile platform for exploring quantum computation and quantum information.

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